UNIVERSITY OF SASKATCHEWAN
ELECTRICAL ENGINEERING 455.3

Assignment Quiz 5
November 7, 1997

Instructor: B.L. Daku
Time: 15 minutes
Note: No aids

(2)

Name:
Student Number:

1. For the following Z-transforms, what are the defining charateristics of the inverse Z-transform. Use only the characteristics from the following list.

   - finite duration
   - infinite duration
   - causal
   - anticausal
   - two-sided

   (a) $X_1(z)$; ROC Entire Z-plane except $z = \infty$.

   causal

   (b) $X_2(z)$; ROC $|z| > r_2$.

   infinite duration ✓

   (c) $X_3(z)$; ROC $r_2 < |z| < r_1$.

   finite duration ✗

   1/2

2. Determine the Z-transform of the sequence $x(n)$ (remember the ROC).

$$x(n) = -r^n e^{j\theta n} u(-n-1)$$

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n} = \sum_{-\infty}^{-1} (-r^n) e^{j\theta n} z^{-n} = \sum_{-\infty}^{-1} (-re^{j\theta})^n z^{-n}$$

$$= \frac{1}{1-(-re^{j\theta})z^{-1}}$$

$$\boxed{X(z) = \frac{1}{1 + re^{j\theta} z^{-1}}}$$

1 1/2

$$ROC: \quad 0 < 1 + re^{j\theta} z^{-1}$$
$$-re^{j\theta} z^{-1} < 1$$
$$-re^{j\theta} < z$$

$$\boxed{|re^{j\theta}| < |z|}$$

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1. Find the inverse z-transform of

$$X(z) = \frac{2z^2 - 3.25z}{(z - 0.25)(z - 3)} \quad ROC\ 0.25 < |z| < 3.$$

$$\frac{X(z)}{z} = \frac{2z - 3.25}{(z - .25)(z - 3)}$$

$$A = \frac{2z - 3.25}{(z - 3)} \Big|_{z=.25}$$
$$= \frac{.5 - 3.25}{-2.75} = 1$$

$$\therefore \frac{X(z)}{z} = \frac{A}{(z - .25)} + \frac{B}{(z - 3)}$$

$$B = \frac{2z - 3.25}{(z - .25)} \Big|_{z=3}$$
$$= \frac{6 - 3.25}{2.75} = 1$$

$$\frac{X(z)}{z} = \frac{1}{(z - .25)} + \frac{1}{(z - 3)}$$

$$\boxed{\therefore x(n) = (.25)^n u(n) + (3)^n u(n)}$$

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2. A system has an input $x(n) = \delta(n) + 3\delta(n-2)$ and an output $y(n) = a^n u(n) + 3a^{(n-2)}u(n-2)$. What is the system function and the impulse response of the system? What values of a will ensure that the system is stable?

$$X(z) = \frac{1}{z} + \frac{3}{z^2}$$
$$= \frac{z^{-2} + 3z}{z^2}$$
$$= z^{-1} + 3z^2$$

$$Y(z) = \frac{1}{1 - az^{-1}} + \frac{3}{1 - az^{-2}}$$
$$= \frac{1 - az^{-2} + 3(1 - az^{-1})}{(1 - az^{-1})(1 - az^{-2})}$$
$$\frac{4 - (az^{-2} + 3z^{-1})}{1 - az^{-2} - az^{-2} + a^2 z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4 - a(z^{-2} + 3z^{-1})}{(1 - az^{-2})(1 - az^{-2})(z^{-1} + 3z^2)}$$ ✗

$$\boxed{|a| < |z|}$$ ✓